2. Basis Step: Check that the formula works for $n = 1$.

$1^2 = \frac{1(1+1)(2 \cdot 1 + 1)}{6} \rightarrow 1 = 1$ ✓

Inductive Step: Assume that

$1 + 4 + 9 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$.

Show that $1 + 4 + 9 + \ldots + k^2 + (k+1)^2$

$= \frac{(k+1)[(k+1)+1][2(k+1)+1]}{6}$.

$1 + 4 + 9 + \ldots + k^2 + (k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$

$= \frac{k(k+1)(2k+1) + 6(k+1)(k+1)}{6}$

$= \frac{(k+1)[k(2k+1) + 6(k+1)]}{6}$

$= \frac{(k+1)(2k^2 + k + 6k + 6)}{6}$

$= \frac{(k+1)(2k^2 + 7k + 6)}{6}$

$= \frac{(k+1)(k+2)(2k+3)}{6}$

$= \frac{(k+1)[(k+1)+1][2(k+1)+1]}{6}$ ✓

3. Basis Step: Check that the formula works for $n = 1$.

$2^{1-1} = 2^1 - 1 \rightarrow 2^0 = 2 - 1 \rightarrow 1 = 1$ ✓

Inductive Step: Assume that

$1 + 2 + 2^2 + 2^3 + \ldots + 2^{k-1} = 2^k - 1$.

Show that $1 + 2 + 2^2 + 2^3 + \ldots + 2^{k-1} + 2^{(k-1)-1}$

$= 2^{k+1} - 1$.

$1 + 2 + 2^2 + 2^3 + \ldots + 2^{k-1} + 2^{(k-1)-1}$

$= (2^k - 1) + 2^{(k+1)-1}$

$= 2^k - 1 + 2^k$

$= 2(2^k) - 1$

$= 2^{k+1} - 1$ ✓

4. Basis Step: Check that the formula works for $n = 1$.

$a_1 \cdot r^{1-1} = a_1\left(\frac{1 - r^1}{1 - r}\right) \rightarrow a_1 r^0 = a_1 \cdot 1 \rightarrow a_1 \cdot 1 = a_1 \cdot 1$ ✓

Inductive Step: Assume that $\sum a_1 \cdot r^{k-1} = a_1\left(\frac{1 - r^k}{1 - r}\right)$.

Show that $\sum a_1 \cdot r^{k+1-1} = a_1\left(\frac{1 - r^{k+1}}{1 - r}\right)$.

$\sum (a_1 \cdot r^{k-1}) + a_1 \cdot r^{k+1-1}$

$= a_1\left(\frac{1 - r^k}{1 - r}\right) + a_1 \cdot r^{k+1-1}$

$= a_1\left(\frac{1 - r^k}{1 - r}\right) + a_1 \cdot r^k$

$= a_1\left[\left(\frac{1 - r^k}{1 - r}\right) + r^k\right]$

$= a_1\left(\frac{1 - r^k + r^k(1 - r)}{1 - r}\right)$

$= a_1\left(\frac{1 - r^k + r^k - r^{k+1}}{1 - r}\right)$

$= a_1\left(\frac{1 - r^{k+1}}{1 - r}\right)$ ✓

5. Basis Step: Check that the formula works for $n = 1$.

$\frac{1}{1(1+1)} = \frac{1}{1+1} \rightarrow \frac{1}{1(2)} = \frac{1}{2}$ ✓

Inductive Step: Assume that

$\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \ldots + \frac{1}{k(k+1)} = \frac{k}{k+1}$.

Show that $\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \ldots + \frac{1}{k(k+1)} +$

$\frac{1}{(k+1)[(k+1)+1]} = \frac{k+1}{(k+1)+1}$.

$\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \ldots + \frac{1}{k(k+1)} + \frac{1}{(k+1)[(k+1)+1]}$

$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$

$= \frac{k(k+2)}{(k+1)(k+2)} + \frac{1}{(k+1)(k+2)}$

$= \frac{k^2 + 2k + 1}{(k+1)(k+2)}$

$= \frac{(k+1)(k+1)}{(k+1)(k+2)}$

$= \frac{k+1}{k+2}$

$= \frac{k+1}{(k+1)+1}$ ✓

6. Basis Step: Check that the formula works for $n = 1$.
$(2 \cdot 1)^2 = \dfrac{2 \cdot 1(1+1)(2 \cdot 1 + 1)}{3} \rightarrow 4 = \dfrac{2(2)(3)}{3} \rightarrow$
$4 = \dfrac{12}{3} \rightarrow 4 = 4$ ✓
Inductive Step: Assume that
$4 + 16 + 36 + \ldots + (2k)^2 = \dfrac{2k(k+1)(2k+1)}{3}$.
Show that $4 + 16 + 36 + \ldots + (2k)^2 + [2(k+1)]^2$
$= \dfrac{2(k+1)[(k+1)+1][2(k+1)+1]}{3}$.
$4 + 16 + 36 + \ldots + (2k)^2 + [2(k+1)]^2$
$= \dfrac{2k(k+1)(2k+1)}{3} + [2(k+1)]^2$
$= \dfrac{2k(k+1)(2k+1)}{3} + \dfrac{3[2(k+1)]^2}{3}$
$= \dfrac{2k(k+1)(2k+1) + 2(2k+2)^2}{3}$
$= \dfrac{2k(k+1)(2k+1) + 3[4(k^2+2k+1)]}{3}$
$= \dfrac{2k(k+1)(2k+1) + 12(k+1)^2}{3}$
$= \dfrac{2k(k+1)[k(2k+1) + 6(k+1)]}{3}$
$= \dfrac{2(k+1)(2k^2+k+6k+6)}{3}$
$= \dfrac{2(k+1)(2k^2+7k+6)}{3}$
$= \dfrac{2(k+1)(k+2)(2k+3)}{3}$
$= \dfrac{2(k+1)[(k+1)+1]2(k+1)+1]}{3}$ ✓

7. A recursive formula for the nth hexagonal number is $H_n = H_{n-1} + 4n - 3$.
Basis Step: Check that the formula works for $n = 1$.
$H_1 = 1(2(1) - 1) \rightarrow 1 = 1(2 - 1) \rightarrow 1 = 1$ ✓
Inductive Step: Assume that $H_k = k(2k - 1)$.
Show that $H_{k+1} = (k+1)[2(k+1) - 1]$.
$H_{k+1} = H_{(k+1)-1} + 4(k+1) - 3$
$= H_k + 4(k+1) - 3$
$= k(2k-1) + 4(k+1) - 3$
$= 2k^2 - k + 4k + 4 - 3$
$= 2k^2 + 3k + 1$
$= (k+1)(2k+1)$
$= (k+1)[2(k+1) - 1]$ ✓

Chapter 13

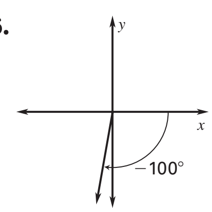
13.1 Skill Practice (pp. 856–857)

3. $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\tan\theta = \frac{12}{5}$, $\csc\theta = \frac{13}{12}$, $\sec\theta = \frac{13}{5}$, $\cot\theta = \frac{5}{12}$ **4.** $\sin\theta = \frac{3\sqrt{73}}{73}$, $\cos\theta = \frac{8\sqrt{73}}{73}$, $\tan\theta = \frac{3}{8}$, $\csc\theta = \frac{\sqrt{73}}{3}$, $\sec\theta = \frac{\sqrt{73}}{8}$, $\cot\theta = \frac{8}{3}$ **5.** $\sin\theta = \frac{8}{11}$, $\cos\theta = \frac{\sqrt{57}}{11}$, $\tan\theta = \frac{8\sqrt{57}}{57}$, $\csc\theta = \frac{11}{8}$, $\sec\theta = \frac{11\sqrt{57}}{57}$, $\cot\theta = \frac{\sqrt{57}}{8}$ **6.** $\sin\theta = \frac{7}{15}$, $\cos\theta = \frac{4\sqrt{11}}{15}$, $\tan\theta = \frac{7\sqrt{11}}{44}$, $\csc\theta = \frac{15}{7}$, $\sec\theta = \frac{15\sqrt{11}}{44}$, $\cot\theta = \frac{4\sqrt{11}}{7}$ **7.** $\sin\theta = \frac{\sqrt{115}}{14}$, $\cos\theta = \frac{9}{14}$, $\tan\theta = \frac{\sqrt{115}}{9}$, $\csc\theta = \frac{14\sqrt{115}}{115}$, $\sec\theta = \frac{14}{9}$, $\cot\theta = \frac{9\sqrt{115}}{115}$ **8.** $\sin\theta = \frac{\sqrt{17}}{17}$, $\cos\theta = \frac{4\sqrt{17}}{17}$, $\tan\theta = \frac{1}{4}$, $\csc\theta = \sqrt{17}$, $\sec\theta = \frac{\sqrt{17}}{4}$, $\cot\theta = 4$ **29. b.** A regular n-gon inscribed in a circle can be divided into n identical triangles, each with side lengths of 1. The angle formed by the two radii of the circle, $\angle A$, has a measure of $\left(\frac{360}{n}\right)^\circ$. When the altitude of the triangle is drawn, $\angle A$ is divided in half so the measure of each half-angle is $\left(\frac{180}{n}\right)^\circ$. The length of the bottom leg of the right triangle is $\sin\left(\frac{180}{n}\right)^\circ$, which is half of one side of the the n-gon. To find the perimeter of the n-gon, n(the number of large triangles) must be multiplied by $2 \cdot \sin\left(\frac{180}{n}\right)^\circ$ (the length of one side of the n-gon), so $P = 2n \cdot \sin\left(\frac{180}{n}\right)^\circ$.

13.3 Skill Practice (pp. 870–871)

3. $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$, $\csc\theta = \frac{17}{15}$, $\sec\theta = \frac{17}{8}$, $\cot\theta = \frac{8}{15}$ **4.** $\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$, $\csc\theta = \frac{5}{4}$, $\sec\theta = -\frac{5}{3}$, $\cot\theta = -\frac{3}{4}$ **5.** $\sin\theta = -\frac{24}{25}$, $\cos\theta = -\frac{7}{25}$, $\tan\theta = \frac{24}{7}$, $\csc\theta = -\frac{25}{24}$, $\sec\theta = -\frac{25}{7}$, $\cot\theta = \frac{7}{24}$ **6.** $\sin\theta = -\frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\tan\theta = -\frac{12}{5}$, $\csc\theta = -\frac{13}{12}$, $\sec\theta = \frac{13}{5}$, $\cot\theta = -\frac{5}{12}$ **7.** $\sin\theta = -\frac{\sqrt{2}}{2}$, $\cos\theta = \frac{\sqrt{2}}{2}$, $\tan\theta = -1$, $\csc\theta = -\sqrt{2}$, $\sec\theta = \sqrt{2}$, $\cot\theta = -1$ **8.** $\sin\theta = \frac{3\sqrt{13}}{13}$, $\cos\theta = -\frac{2\sqrt{13}}{13}$, $\tan\theta = -\frac{3}{2}$, $\csc\theta = \frac{\sqrt{13}}{3}$, $\sec\theta = -\frac{\sqrt{13}}{2}$, $\cot\theta = -\frac{2}{3}$ **9.** $\sin\theta = -\frac{5\sqrt{34}}{34}$, $\cos\theta = -\frac{3\sqrt{34}}{34}$, $\tan\theta = \frac{5}{3}$, $\csc\theta = -\frac{\sqrt{34}}{5}$, $\sec\theta = -\frac{\sqrt{34}}{3}$, $\cot\theta = \frac{3}{5}$ **10.** $\sin\theta = -\frac{\sqrt{11}}{6}$, $\cos\theta = \frac{5}{6}$, $\tan\theta = -\frac{\sqrt{11}}{5}$, $\csc\theta = -\frac{6\sqrt{11}}{11}$, $\sec\theta = \frac{6}{5}$, $\cot\theta = -\frac{5\sqrt{11}}{11}$

16.



17.